The forty members of your school adventure club are trying to decide what type of trip to take. The chart shows how the club members rank the three options.

	Number of Voters					
Ranking	**10**	**7**	**1**	**10**	**4**	**8**
First choice	skiing	skiing	rafting	rafting	caving	caving
Second choice	rafting	caving	skiing	caving	skiing	rafting
Third choice	caving	rafting	caving	skiing	rafting	skiing

1. A common method of voting is *plurality*. In this system, each person casts one vote for a first choice and the option with the most votes wins. On the basis of the chart, which activity would be the winner under the plurality system? _____

2. Which activity is liked least by the largest number of members; that is, which activity is ranked third by the largest number of voters? _____

3. Why might the plurality method not produce results satisfactory to all voters?

4. Why do you think the plurality method is used so often?

5. Think of some variations of plurality voting or other voting techniques that might prove more satisfactory to the voters. Within your group, describe or develop at least two other vote-tallying methods that have not yet been discussed in class.

Use the set of preference lists (chart) from the first part of this activity sheet to answer the following:

6. The *Hare voting system* involves taking an initial poll in which each person casts one vote for his or her favorite option. The option receiving the least number of first-place votes is eliminated, and another poll is taken. Those members who originally voted for the eliminated option vote for their second choice in the runoff election. Continue eliminating the options that receive the fewest votes and repolling until a single winner or tied winners remain. Which activity would the adventure club choose using the Hare system? _____ Describe the process as options are eliminated.

From the *Mathematics Teacher*, January 2000

7. Another voting method, called a *Borda count,* takes into account each voter's first, second, and third choices. Each first-choice vote is awarded two points, each second-choice vote is worth one point, and third choices receive no points. For example, skiing has seventeen first-choice votes and five second-choice votes, for a total of 2(17) + 1(5), or 39, points. Determine the total number of points for the other two activities. Which activity has the most points using this method?

8. *Sequential pairwise voting* involves a sequence of head-to-head contests. The organization first votes on two of the options, and the preferred option is then matched with the next option while the "loser" is eliminated. A club member suggests that the club should first vote between skiing and caving and then have a vote between the winner of that contest and the remaining option, rafting. Which activity is chosen by this method?

9. Which of the methods—plurality, Hare, Borda count, or sequential pairwise voting—is the fairest in this situation? Why? Which is the least fair? Why?

10. Suppose that your preference is rafting. Devise a voting system that would enable rafting to be chosen and that would sound fair to the other club members.

From the *Mathematics Teacher,* January 2000

(Continued on page 27)

(Continued from page 23)

1. Suppose that you are one of the ten sports editors whose votes together determine the rankings of state high school football teams. A variation of the Borda count is used in which each voter ranks his or her top ten teams out of the many teams in the division. Points are awarded as follows:

   10 points for each first-place vote
   9 points for each second-place vote
   ⋮
   1 point for each tenth-place vote

   You suspect that the other nine sports editors will rank Big City High first and your high school's team second. Can you submit a ranking that will enable your team to earn the highest number of points? Explain.

2. Candidates A, B, and C are running for office. The preferences of the voting community are given in the following chart:

	Percent			
Ranking	**38%**	**29%**	**24%**	**10%**
First choice	A	B	C	A
Second choice	B	A	B	C
Third choice	C	C	A	B

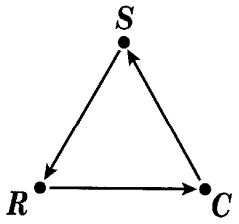
   a) Who will win the election if the plurality method is used? _____

   b) By still using the plurality method, does a strategy exist that the voters in the 24 percent column could use to get a result more to their liking?

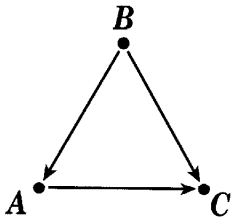
   c) Who will win the election if the Hare system is used? _____ Which column of voters would be most disappointed in the result? _____ Devise an alternative ranking that they could submit to get a result more to their liking when the Hare system is used.

We can use a diagram called a *tournament digraph* to illustrate the expected results of head-to-head polls among the candidates. We use vertices (dots) to represent each of the candidates and draw an arrow from one candidate to another if the first candidate would beat the second in a head-to-head competition. If no ties occur, exactly one arrow will join every pair of candidates.

Example: The tournament digraph corresponding to the situation on the first sheet is sketched at the right. Note that in head-to-head matches, skiing beats rafting, rafting beats caving, and caving beats skiing.



1. In the example above, no activity defeated all the other activities in head-to-head voting. Suppose instead that the tournament digraph for a set of preference lists looked liked this:



Who would you expect to win using plurality? _____

Hare system? _____

Borda count? _____

Sequential pairwise voting? _____

2. Sketch the tournament digraph for the election in problem 2, sheet 2. Does this result agree with your claim in question 1?

3. How many arrows are in a tournament digraph with five candidates? _____ With ten candidates? _____ With n candidates? _____ Can you develop a formula? _____

4. Sketch the tournament digraph for the following:

	Number			
Ranking	**3**	**2**	**2**	**2**
First choice	W	Z	X	X
Second choice	X	Y	Z	W
Third choice	Y	W	Y	Y
Fourth choice	Z	X	W	Z

5. Determine the winner of the election in problem 4 when the Borda count method is used. _____ Is it what you expected? _____ Note that with four candidates, each first-place vote will be worth 3 points, each second-place vote will be worth 2 points, and each third-place vote will be worth 1 point.

6. When a candidate beats every other candidate in head-to-head contests, we call the candidate a *Condorcet winner.* Develop a table of preference lists for which the Condorcet winner would lose the election under the Hare system of voting.

7. Is it possible to have a Condorcet winner who does not win using the sequential pairwise voting method? _____ Support your answer.

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